Math 304 Sample Midterm 2

Question 1 Show that for functions $f, g : \mathbb{R}^3 \to \mathbb{R}$, $\operatorname{div}(\nabla f \times \nabla g) = 0$.

Question 2 Find the area of the region D in the first quadrant bounded by the parabolas $y = x^2$, $y = 2x^2$, $x = y^2$, and $x = 4y^2$, by using the transformation $T : D^* \to D$ given by $x = (u^2 v)^{-1/3}$, $y = (uv^2)^{-1/3}$.

Question 3 Evaluate the path integral $\int_{\mathbf{c}} f(x, y, z)\, ds$, where $f(x, y, z) = 2x + 9xy$, and $\mathbf{c}(t) = (t, t^2, t^3)$, $0 \leq t \leq 2$.

Question 4
a) Evaluate the line integral $\int_{\mathbf{c}} x^2 y\, dx + xy^3\, dy$, where $\mathbf{c}$ consists of the line segments from $(0, 0)$ to $(3, 3)$ and from $(3, 3)$ to $(0, 3)$.
b) Evaluate the line integral $\int_{\mathbf{c}} (e^y + ye^x)\, dx + (e^x + xe^y)\, dy$, where $\mathbf{c}$ is the part of the graph $y = \ln 6x$ joining $(1/6, 0)$ to $(1/2, \ln 3)$.

Question 5
a) Find the equation of the tangent plane to the parametrized surface S in $\mathbb{R}^3$ given by $(u, v) \mapsto (u^2 - v^2, u + v, u^2 + 4v)$ at the point $(-1/4, 1/2, 2)$.
b) Use spherical coordinates to find the area of the surface on the sphere $x^2 + y^2 + z^2 = 4$ that lies above the cone $z = \sqrt{x^2 + y^2}$.

Question 6 Evaluate $\iint_S z\, dS$, where S is the surface whose side S_1 is given by the cylinder $x^2 + y^2 = 1$, whose bottom S_2 is the disk $x^2 + y^2 \leq 1$, and whose top S_3 is part of the plane $z = (1 + x)$ that lies above S_2.